Assessing students' mathematical skills, knowledge and understanding

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This presentation

Teachers need to recognise the mathematical skills, knowledge and understandings that their students bring to the classroom.

Mathematics assessment is integral in enabling teachers to determine what their students know, the misconceptions they have developed and the difficulties they may be experiencing when presented with a mathematics task.

In this presentation, two mathematics assessment instruments will be discussed: Newman's Prompts for worded problems and the use of the clinical interview e.g. The Mathematics Online Interview.

These assessment instruments allow teachers to gather information about whether a student understands what the task requires them to do, the strategies the students use to solve the task, the vocabulary they use to describe their strategy and whether they can complete the calculations needed for successful completion of the task. Information from these instruments allows teachers to provide the learning opportunities that build on their students' mathematical skills, knowledge and understandings.

Consider the language of this task

Content Domain	Main Topic	Cognitive Domain
NUMBER	Whole Numbers	Reasoning

Place the four digits 3, 5, 7, and 9 into the boxes below in the positions that would give the greatest result when the two numbers are multiplied.

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Place the four digits 3, 5, 7, and 9 into the boxes below in the positions that would give the greatest result when the two numbers are multiplied.

$$\begin{array}{r} \square\ \square \\ \times\ \square\ \square \\ \hline \end{array}$$

34% Australian Year 8 students gave correct answer

Newman's prompts

Ann Newman's prompts

Newman's prompts

Newman suggested **five prompts** to determine where errors may occur in students' attempts to solve written problems.

The Hurdles	Newman's Prompts
Reading	1. Please read the question to me.
Comprehension	2. Tell me what the question is asking you to do.
Transformation	3. Tell me how you are going to find the answer.
Processing Skills	4. Show me what to do to get the answer.
Encoding	5. Now, write down your answer.

Solve the following:

Natalie paddled 402 km of the Murray River in her canoe over 6 days. She paddled the same distance each day. How far did Natalie paddle each day?

In groups discuss the steps you took to solve the problem.

Ann Newman's Prompts

Newman's Prompts	Example problem
1. Read the question	*Natalie paddled 402 km of the Murray River in her canoe over 6 days. She paddled the same distance each day. How far did Natalie paddle each day?*
2. What is the question asking you to do?	
3. How are you going to find the answer?	
4. Do the working out.	
5. Write the answer to the problem.	

Solve using Ann Newman's prompts

Explore the different arrangements of the digits 2, 4, 6, 8 and 9 so that you can create the largest product possible, using only one multiplication sign. Each digit must be used, and can only be used once.

Discuss:

Did you use the prompts?

Did you find them useful?

An example for younger students

Newman's Prompts	Example problem
1. Read the question	Danny has 8 cars. Mark has three times as many cars as Danny. How many cars does Mark have?
2. What is the question asking you to do?	Find out how many cars Mark has
3. How are you going to find the answer?	8 x 3 or 8 + 8 + 8 =
4. Do the working out	8 and 8 are 16 and 8 more makes 24.
5. Write the answer to the question	Mark has 24 cars.

https://education.nsw.gov.au/teaching-and-learning/student-assessment/smart-teaching-strategies/numeracy/number-and-algebra/multiplication-and-division/S2-multiplication-and-division

Anne Newman's Prompts

Design an open-ended word problem to use with students.

What would you expect them to say for prompts 2 – 5?

Newman's prompts

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Encoding	5. Now, write down your answer.

Discuss

Would the five prompts be useful for students at your school /in your class?

- Why?

- Why not?

Using an interview for assessment

What is an interview?

- a dialogue between an adult interviewer and student

- goal is to understand student's thinking

- dialogue centred around a standard problem or task

- follow up questions depend on student's responses (adaptable)

- interpretations of student thinking depends on skill of interviewer

- yields information not easily accessible from other sources such as paper-and-pencil tests.

- primary goal is to explore limits of student's thinking.

- process of thinking considered more important than correct solution.

Preparing for an interview

- Ensure you are comfortable with the wording of the tasks.

- How and when are you going to rephrase the question?

- What responses do you expect for each task?

- What will you do if you get an unexpected response?

- What will you do if a student gets stressed?

Difficulties with interviews

- Students' expectations may be influenced by the fact that the interviewer may be a relative stranger.

- If interview at school students assume that it involves some kind of test that counts towards an evaluation.

- Students think tasks likely to have right and wrong answers and certain methods approved.

- Interview may take place when students are alert, tired, hungry, distracted or excited.

- Students might prefer to be back in classroom or might prefer break from classroom

Administration of an interview

- need a quiet place where there will be no interruptions

- make the student feel 'at ease'

- explain the need for any equipment

- encourage students to use whatever methods or materials that seem appropriate to them.

- hints or prompts (not teaching) given if necessary

- remind the student to talk out loud about what they are doing or thinking

- accept all responses

- follow-up questions should be given with no indication of 'correctness'

The Mathematics Online Interview

- One-on-one interview away from the regular classroom

- Mainly hands-on tasks incorporating concrete materials

- Focus is on mental computation

- Responses focus on strategies that the students use … not only the correct answer

- 61 questions and sub-questions

- Should take 30 - 40 minutes
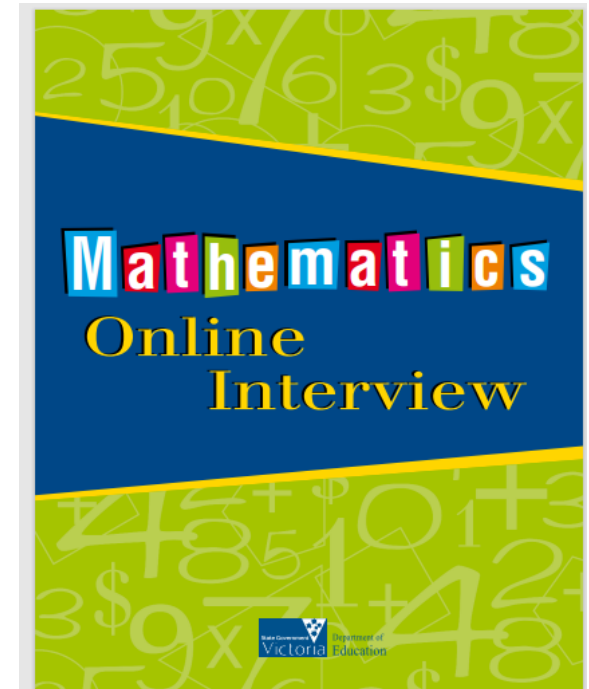
Mathematics Online Interview

The full interview involve about sixty tasks, some with sub-tasks, although students will not attempt all of them.

- While the student is successful, the interviewer continues to the next task in the section

- If the student has difficulty, detour to another question which may define more clearly the child's understanding

- If no detour, then move to next section of the interview

Topics assessed by the Interview

- Counting
- Place value
- Strategies for addition & subtraction
- Strategies for multiplication & division

- Time
- Length
- Mass
- Properties of shape
- Visualisation

Example from the Interview: Detour

I Simpler Counting Tasks / More or Less / Conservation

Place a pile of 20 teddies in front of the child in a scattered pattern, made up of exactly 4 yell...
teddies, 5 red teddies, 3 green teddies, and 8 blue ones.

a) Please put the yellow teddies together.

b) How many yellow teddies are there?

Put a group of 3 green teddies together near the 4 yellow teddies (two different small groups).

c) Are there more green teddies or more yellow teddies?

Push the yellow and green teddies aside.

d) Please get five blue teddies.

e) Now put them in a line. (If the child has already put them in a line, ask the child to 'move them together now'.) . . . Tell me how many blue teddies there are.

This question is part of the Detour designed for students in their first year of school.

Observation Notes

QUESTIONS	OBSERVATIONS
I. Simpler Counting Tasks / More or Less / Conservation *Place a pile of 20 teddies in front of the child in a scattered pattern, made up of exactly 4 yellow teddies, 5 red teddies, 3 green teddies, and 8 blue ones.* a) Please put the yellow teddies together. b) How many yellow teddies are there? *Put a group of 3 green teddies together near the 4 yellow teddies (two different small groups).* c) Are there more green teddies or more yellow teddies? *Push the yellow and green teddies aside.* d) Please get five blue teddies. e) Now put them in a line. *(If the child has already put them in a line, ask the child to 'move them together now'.) … Tell me how many blue teddies there are.*	**Counting Task** • Are they able to gather together the yellow teddies? • Do they count them one by one or are they able to recognise a collection of four without counting. **More or Less Task** • Do they count each group? • Do they line the teddies up next to each other? **Conservation Task** • Note: It is more challenging for students to make a collection of a given amount rather than just counting a collection. • Can they conserve number? i.e. can they automatically state how many teddies there are without recounting them after they have been rearranged?

Example task from the Interview

19 Counting Back

For this question you need to listen to a story.

a) Imagine you have 8 little biscuits in your play lunch and you eat 3.
    How many do you have left? . . . How did you work that out?

If incorrect answer, ask part (b):

b) Could you use your fingers to help you to work it out? *(it's acceptable to repeat the question, but no further prompts should be given).*

😐 ➡ **Section D**

Example task from Visualisation section

60 Design

Show the child the white page with the design on it.

a) *(Pointing to the small pieces on the page)* Which piece is **not** part of the design? This may take a little while.

(If some time passes without comment or action) Would you like to tell me what you are thinking?

b) *(Once the child has decided which piece is not part of the design)* Explain how you know.

Design
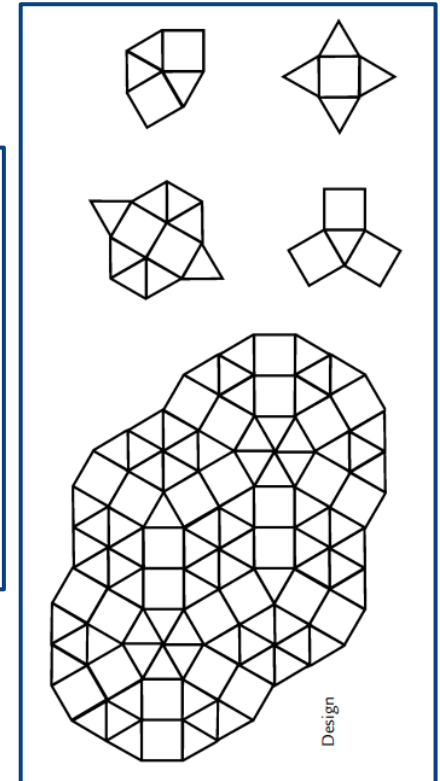
Why use an interview?

Mathematics Online Interview: Counting task

2 Counting Forwards, Backwards, and Breaking the Sequence

Please count for me by ones without the teddies. Start counting from … I'll tell you when to stop.

a) 1 … 32 😐 → part (e)

b) 53 … 62 😐 → part (d)

c) 84 … 113 *(remember to note first difficulty)*

d) Count backwards from 24. I'll tell you when to stop *(24, 23, . . ., 15)*.

   (If child hesitates, say 'like 24, [pause] 23, . . .') 🙂 → Q3 😐 → part (e)

e) Count backwards from 10. I'll tell you when to stop. → **Section B**
   (If child hesitates, say 'like 10, [pause] 9, . . .')

Purpose?
Possible responses?

Record sheet

2. **Forwards/backwards/breaking sequence**

a. 1 → 32 ☐ _____ (1st difficulty)

b. 53 → 62 ☐ _____

c. 84 → 113 ☐ _____

d. 24 → 15 ☐ _____

e. 10 → 1 ☐ _____

Mathematics Growth Points: Counting

0. Not apparent.
   Not yet able to state the sequence of number names to 20.

1. Rote counting
   Rote counts the number sequence to at least 20, but is not yet able to reliably count a collection of that size.

2. Counting collections
   Confidently counts a collection of around 20 objects.

3. Counting by 1s (forward/backward, including variable starting points; before/after)
   Counts forwards and backwards from various starting points between 1 and 100; knows numbers before and after a given number.

Mathematics Growth Points: Counting

4. Counting from 0 by 2s, 5s, and 10s
Can count from 0 by 2s, 5s, and 10s to a given target.

5. Counting from x (where x >0) by 2s, 5s, and 10s
Given a non-zero starting point, can count by 2s, 5s, and 10s to a given target.

6. Extending and applying counting skills
Can count from a non-zero starting point by any single digit number, and can apply counting skills in practical task

Moving from one growth point to the next

Working Out What Works

Working Out What Works (WOWW) Training and Resource Manual:
A teacher professional development program designed to support teachers to
improve literacy and numeracy outcomes for students with learning difficulties
in Years 4, 5 and 6 (2nd edition)

Australian Government
Department of Education, Science and Training

ACER

Growth Points	Incremental steps
1. Counting	
0. Not apparent Not yet able to state the sequence of number names to 20.	When children start to count they recite the verbal sequence, either successfully or with partial success but do not realise that counting is done for a purpose. For these students counting is a recitation of number names and is dependent on their memory. Suggested sequence for verbal counting: • Count by ones forwards to 5 • Count by ones forwards to 10 • Number after (up to 10) • Number before (up to 10) • Number between (up to 10) • Count by ones forwards to 19 • Number after (up to 19) • Number before (up to 19) • Number between (up to 10)

Meaningful or rational counting

According to Gelman & Gallistel *meaningful* or *rational* counting relies on children knowing:

how to count:

- One-to-one principle
- Stable-order principle
- Cardinal principle

what to count

- Abstraction principle
- Order-irrelevance principle

Gelman & Gallistel (1978) *The Child's Understanding of Number*. Cambridge, MA: Harvard University Press)

What strategies would your students use?

23 **Multi-digit Strategies**

I am going to show you some questions. Tell me the answer.

Show the white cards for the following questions [at **any stage**] 😐 ➡ **D**

a)   68 + 32

b)   25 + 99

c)   100 − 68

What to do after the interview?

- analyse the information recorded during the interview and determine the growth points (https://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/assessment/Pages/mathsgrowthpoints.aspx)

- Is there data that doesn't fit with your understanding/ perception of the student's prior knowledge? If so, review the Interview results and consider what might have affected the student's responses.

- maintain a student profile, based on this analysis

- observations, anecdotal notes, monitoring and assessment data collected during mathematics sessions can be used to support and expand the interview-based profiles.

Other interview protocols

Schedule of Early Numeracy Assessment (NSW, NT)
http://assessmentandadjustment.weebly.com/uploads/2/3/7/0/23709518/entiresenakitp1-34.pdf
- assesses current levels of thinking in mathematics
- SENA 1 (Transition – Year 1) and SENA 2 (Years 2 – 3)
- provides teachers with information on students' understandings of number formation and strategies used to solve problems

The Diagnostic Interview (New Zealand)
https://nzmaths.co.nz/sites/default/files/Numeracy/2008numPDFs/NumBk2.pdf
- designed to help teachers understand the Number Framework
- administered as individual interview and assesses student knowledge in 5 knowledge domains and 3 strategy domains.
- It was expected that once a teacher understood the Number Framework that the assessment of a student's strategy stage would be made from in-class observations

The Diagnostic Interview (New Zealand)

Task (6): Find 53 – 26.

Actions: Show the card with the bus problem written on it.

Say: **There are 53 people on the bus. 26 people get off. How many people are left on the bus?**

Decision: If the student imagined using a standard written method, read the notes at the end of task (8). If the student failed to solve this problem correctly, rate her/him at stage 5 and proceed to form B.

Task (7): Find 394 + 79.

Actions: Show the card with the stamp problem written on it.

Say: **Sandra has 394 stamps. She gets another 79 stamps from her brother. How many stamps does she have then?**

Decision: If the student gets both of tasks (6) and (7) correct using part-whole strategies, proceed to task 8. Otherwise rate the student at stage 5 and proceed to form B.

Becoming an Interviewer

Becoming an interviewer

Groups of 3:

- interviewer

- interviewee

- observer/recorder

Change roles:

interviewer ➡ interviewee ➡ observer/recorder

Becoming an interviewer

Interviewer

- **Interview** your colleague following the given script.

- **Encourage** your partner to explain their methods & thinking as they work on the task.

- **Don't collaborate** or teach!

Interviewee

- **Explain** your methods, strategies and thinking to the interviewer.

Observer/Recorder

- **Record** observations – methods, strategies, interesting statements …

- **Share** observations.

Interview Task 1

16 Sorting the Capital Cities

Show the child the pink chart with population figures.

Here is a list of Australia's capital cities *(point to the names of the cities)*. . . . These numbers show how many people live in each city.

[at any stage] 😐 ➔ C

a) *(point to the word Darwin)* How many people live in Darwin?

b) *(point to the word Canberra)* How many people live in Canberra?

c) *(point to the word Adelaide)* How many people live in Adelaide?

d) Please point to the city that has the third largest number of people.

e) How did you work that out?

Capital City	Population (June 1998)
Adelaide	1 088 349
Brisbane	1 574 615
Canberra	308 086
Darwin	86 576
Hobart	194 974
Melbourne	3 371 308
Perth	1 341 914
Sydney	3 986 723

39

Interview Task 2

26 **Estimating and Calculating Subtraction**

Show the yellow card with 642 – 376.

a) Please read this to me.

b) Please estimate the answer to this *(If necessary, prompt:* 'what would the answer be "round about"?'*)*

> **No estimate or one outside the range 200–300 ➜ Section D**

c) Can you work out the exact answer to this in your head? *(266)*

If 'yes' (unlikely!), encourage the child to try to do so. If not successful (or if the response to the previous question is 'no'), make the following request:

Please use the paper to work it out **any way you like**.

> **➜ Section D**

Interview Task 3

37 **Missing Number**

Show the orange card with $54 \times$ ___ $=$ ___ ___ 2

a) The answer to $54 \times$? ends in 2. What can you tell me about this missing number?
   (Pointing to the space after the multiplication sign.) 😐 ➜ **E**

b) How did you work that out?

c) Could it be any other number? How do you know?

Becoming an Interviewer

Reflections on being the Interviewer:

- What did you do well?

- What would you try to improve next time?

Reflections on being interviewed:

- How did you feel as you were completing the tasks?

Reflections on being observer/recorder

- What did you do well?

- What would you try to improve next time?

Creating interview tasks

TIMSS example

Exhibit 2.8 **TIMSS 2007 High International Benchmark (550) of Mathem...** Example Item 3

Content Domain: Number

Description: Determines the missing digit to give a specified difference in a three-digit subtraction problem.

```
 942
-5■7
----
 415
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Mano did the subtraction problem above for homework but spilled some of his drink on it. One digit could not be read. His answer of 415 was correct. What is the missing digit?

Answer: ___2___

Country	Percent Full Credit	
Chinese Taipei	88 (1.6)	◉
Hong Kong SAR	85 (1.9)	◉
Singapore	85 (1.4)	◉
Russian Federation	84 (1.8)	◉
¹ Kazakhstan	83 (3.1)	◉
Japan	80 (1.8)	◉
¹ Lithuania	71 (2.3)	◉
¹ Latvia	71 (2.6)	◉
Ukraine	68 (2.3)	◉
Armenia	66 (3.0)	◉
¹ Georgia	60 (2.7)	◉
Hungary	51 (2.8)	◉
Slovak Republic	50 (2.3)	◉
Italy	49 (2.1)	◉
International Avg.	42 (0.4)	
Germany	41 (2.2)	
Czech Republic	41 (2.6)	
² † United States	41 (1.8)	
Austria	41 (2.4)	
Slovenia	31 (2.0)	◉
‡ Netherlands	31 (2.6)	◉
Iran, Islamic Rep. of	29 (2.2)	◉
† Denmark	28 (2.5)	◉
England	28 (2.1)	◉
Colombia	25 (2.1)	◉
† Scotland	25 (2.2)	◉
Australia	22 (2.6)	◉
Sweden	18 (1.7)	◉
New Zealand	18 (1.6)	◉
Norway	18 (1.9)	◉
Tunisia	18 (1.8)	◉
Algeria	16 (1.9)	◉
Morocco	14 (1.7)	◉
El Salvador	13 (1.6)	◉
◄◄ Kuwait	10 (1.4)	◉
Yemen	7 (1.3)	◉
Qatar	5 (0.8)	◉

TIMSS example

Exhibit 2.13 **TIMSS 2007 Intermediate International Benchmark (475) o Achievement – Example Item 7**

Content Domain: Number

Description: Identifies a three-digit number described in units, tens, and hundreds.

Which number equals 3 ones + 2 tens + 4 hundreds?

(A) 432

● 423

(C) 324

(D) 234

Country	Percent Correct	
Chinese Taipei	89 (1.4)	○
‡ Netherlands	88 (1.8)	
Singapore	86 (1.5)	○
Germany	84 (1.5)	○
England	84 (1.8)	○
Japan	83 (1.6)	○
Hungary	82 (2.2)	○
Russian Federation	82 (1.8)	○
Hong Kong SAR	81 (2.0)	○
¹ Latvia	81 (2.2)	○
Slovak Republic	81 (1.7)	○
↑ Denmark	80 (2.0)	○
Austria	80 (1.7)	○
Sweden	80 (1.6)	○
² ↑ United States	79 (1.4)	○
↔ Kuwait	76 (1.8)	○
Algeria	75 (2.2)	○
¹ Lithuania	73 (2.1)	
↑ Scotland	73 (2.3)	
Slovenia	73 (2.0)	
¹ Kazakhstan	73 (3.3)	
Czech Republic	71 (2.3)	
International Avg.	71 (0.4)	
New Zealand	70 (2.0)	
Italy	69 (2.2)	
Norway	68 (2.4)	
Ukraine	67 (2.4)	
Australia	67 (2.5)	
Iran, Islamic Rep. of	67 (2.4)	
Morocco	65 (2.8)	⊛
Qatar	60 (1.3)	⊛
Tunisia	59 (2.6)	⊛
Armenia	53 (2.5)	⊛
¹ Georgia	50 (3.0)	⊛
Yemen	48 (2.4)	⊛
El Salvador	20 (2.0)	⊛
Colombia	20 (2.0)	⊛

Examples of NAPLAN items

20 These biscuits are sold in packets of 10. Shelley wants to give one biscuit to each of her 27 classmates.

What is the **least** number of packets that Shelley needs?

1	2	3	4
○	○	○	○

Years 3 & 5 2010

12 A flea can jump up to 200 times its body length.
The body length of the flea is 2.5 mm.

What is the furthest distance the flea can jump?

5 mm	50 mm	500 mm	5000 mm
⬭	⬭	⬭	⬭

Year 7 NAPLAN non-calculator (2010)

24 A grocer buys 25 boxes of melons.
Each box costs $28.
The total cost of the boxes is $28 × 25.

Which calculation is another way of working out the total cost?

7×100	18×250	$(56 \div 2) \times 50$	$8 + (20 \times 25)$
◯	◯	◯	◯

Year 7 NAPLAN non-calculator (2010)

I used to think but now I think …

Thank you